RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [2015-18]

B.A./B.Sc. THIRD SEMESTER (July – December) 2016

Mid-Semester Examination, September 2016

Date : 15/09/2016 **MATHEMATICS GENERAL FOR ECONOMICS**

Time : 12 noon – 1 pm Paper : III Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

(*Answer <u>any three</u> questions*) [3×5]

1. If f(x,y) is differentiable at (a,b) prove that the partial derivatives $f_x(a,b)$ and $f_y(a,b)$ exist. Use it to show that $f(x,y) = |x| + |y|$ is not differentiable at (0,0) [3+2]

2. State the Euler's theorem on Homogeneous function of two variables. Use it to prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 y}{\partial y^2} = n(n-1)u$, where u is a differentiable homogenous function of degree n in two independent variable x and y and all partial derivatives of first and 2nd order are continuous. [1+4]

3. Show that for the function

$$f(x,y) = \begin{cases} (x^2+y^2)\tan^{-1}\frac{y}{x} & : \quad x \neq 0 \\ \frac{\pi}{2}y^2 & : \quad x = 0 \end{cases},$$

$f_{xy}(0,0) \neq f_{yx}(0,0)$. [5]

4. Let $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$. Prove that $\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^2 u = -\frac{\sin u \cos 2u}{4\cos^3 u}$. [5]

Group – B

5. Answer **<u>any two</u>** questions : [2×3]

   a) Find the differential equation for $u = B + \frac{A}{r}$, where A and B are any arbitrary constants.

   b) Find the particular solutions of $\cos y\,dx + (1+2e^{-x})\sin y\,dy = 0$ when x = 0, $y = \frac{\pi}{4}$.

   c) Solve : $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2+y^2} = 0$.

6. Answer **<u>any one</u>** question : [1×4]

   a) Solve : $x\frac{dy}{dx} + y = x^2y^2$.

   b) Obtain the complete and singular solution for the differential equation $y = px + p - p^2$, $p \equiv \frac{dy}{dx}$.

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